## KS5 Mathematics



## KS5 Maths

## STUDENT HANDBOOK

## Name

Good mathematics is not about how many answers you know....It's how you behave when you don't know. A

## KS5 Mathematics

## Dear student of mathematics,

Thank you for choosing to study maths here at UTC Swindon!
Maths is one of the most vital subjects there is to learn because of its applications in real life, especially in engineering and computer science courses.

The scale of mathematics embedded in engineering and computer courses is enormous, and here at UTC Swindon we acknowledge that for you to be able to succeed in your courses, it is crucial that you develop your mathematical skills.

In this student handbook you will find all information about the KS5 maths courses here at UTC Swindon. At the end of the handbook, you will find a maths task called 'Bridge to KS5 Maths, Problem Solving'; we would appreciate it if you could attempt to complete as many questions as you can before starting your course in September. The 'Bridge to KS5 Maths, Problem Solving' task contains all the prior maths topics that you will need to master from your GCSE maths course, for you to start your journey at UTC Swindon, as a brilliant mathematician with a solid maths background.

After reading the information provided in this handbook, and if you still have any questions, please email the Head of Mathematics, Dani Amorim: danamorim@utcswindon.co.uk

## Handbook contents

- Rationale
- Expectations
- Curriculum outline
- Evidence of work
- Assessments
- Fun reading list
- Command and key words for KS5
- Mathematical notations
- Task: ‘The Bridge to KS5 Maths - Problem Solving'


## Rationale

'Mathematics is essential for everyday life and understanding our world. It is also essential to science, technology and engineering, and the advances in these fields on which our economic future depends. It is therefore fundamentally important to ensure that all pupils have the best possible mathematics education. They need to understand the mathematics they learn so that they can be creative in solving problems, as well as being confident and fluent in developing and using the mathematical skills so valued by the world of industry and higher education.'

Sir Michael Wilshaw
Her Majesty's Chief Inspector

## Expectations

What we observe is not nature itself, but nature exposed to our method of questioning.

Werner Heisenberg

1. Punctuality.
2. Acceptable Classroom Conduct.
3. Good Organisation.
4. Homework.
5. Independent Work.


## Curriculum Outline

A@A UTC Swindon Maths Department follows the AQA GCE Mathematics Specification (AS 7356 and A-level 7357).


Geometry


Exponentials and logarithms

Statistics


Calculus


Sequences and series



Mechanics

## AQA

AS and A-level Mathematics two-year specification content:


Algebra and proof


Trigonometry


Numerical methods

Below are some of the requirements and informations from AQA and the Department of Education regarding the maths courses that we provide here at UTC Swidon:

## Use of data in statistics:

The Department of Education (DfE) have set out the following requirements regarding the use of data in statistics as follows:

AS and A-Level mathematics must require students to:

- Became familiar with one or more specific large data set(s) in adbance of the final assessment (these data must be real and sufficiently rich to enable the conceprts and skills of data presentation and interpretation in the specifiaction to be explored)
- Use technology such as spreadsheets or specialist statistical packages to explore the data $\operatorname{set}(\mathrm{s})$
- Interpret real data presented in summary or graphical form
- Use data to investigate questions arising in real contexts

Specification will require student to explore the data set(s), and associated contexts, during their course or study to enable them to perform tasks that assume familiarity with the contexts, the main features of the data and the ways in which technology can help explore the data. Specifications should also
require students to demonstrate the ability to analyse a subset or features of the data using a calculator with standard statistical functions.

## 'Synopticity' and 'problem solving':

One of the requirements of the A-level specification is to test the content synoptically and for students to apply the knowledge they have in unfamiliar areas. In section four of the specification, it says that students should be able to 'draw together information from different areas of the specification' and 'apply their knowledge and understanding in practical and theoretical contexts'. The effect of these requirements is that, to assess a student's ability to bring together the different elements of their mathematical knowledge, we need to allocate a certain number of marks to problems which are, unpredictable. We can ask students to use any techniques in the specification to solve problems.

Teachers are advised to give students opportunities to solve unfamiliar problems based on well embedded knowledge throughout the course so that when faced with unconventional questions in the exam they are prepared to attempt it. We are conscious that synoptic questions can be more demanding than others. We must include one synoptic question on each exam paper, but we are very careful to make questions as accessible as possible. It can often be the case that 'problems' are more demanding than routine questions, but as we need to allocate $25 \%$ ( $20 \%$ at AS) of marks to problem solving and modelling assessment objectives, we work hard to make these questions as accessible as we can.

There is strong research evidence that students are best placed to solve problems when they have expert knowledge of the maths required. For teachers, this suggests that setting demanding problems on topics as you teach them is not likely to be effective in developing a positive approach to solving problems. Beginning the course with an element of problem solving based on well understood GCSE Maths is more likely to be effective.

## Calculators:

Using calculators in exams is more important now than it was in the previous modular specification, and we really embrace this development. Therefore, you must have a suitable calculator for A-Level courses. Here's a list of some of the more common things we expect students to be able to do with a calculator in exams:

- solve quadratic equations
- find the coordinates of the vertex of a quadratic function
- solve quadratic inequalities
- solve simultaneous linear equations in two variables
- calculate summary statistics for a frequency distribution
- repeat an iterative process, including the Newton-Raphson method - find binomial and normal probabilities and find the z -value for a normal distribution
- calculate a definite integral
- calculate the derivative of a function at a given point
- solve equations when the question permits, e.g., when problem solving and modelling.

Generally, if a calculator can be used then we expect students to do so and there will be no extra credit for using a handwritten method. The specification says that students should "use technology such as

## KS5 Mathematics

calculators and computers effectively and recognise when such use may be inappropriate'.' Consequently, students should be aware of the range of functions available on their calculator as well as their limitations.

Calculators will not be appropriate in every situation.

- We will include parameters in some questions so that they cannot be completed on a calculator when we need to test students' abilities to carry out particular techniques.
- 'Exact value' means that the answer will often involve surds, e, or $\pi$ and should not be given in decimal form.
- The instructions 'Show that', 'Prove' and 'Fully justify' mean we require a method with all steps clearly shown.

Calculators may be used to help with intermediate steps but the input and output should be written down and the working must meet the requirements of the question.

## Mark scheme:

We've redesigned our mark schemes to help teachers and students to fully understand what's expected in order to gain marks. The mark scheme is designed to be applied positively and to reward students for making progress even if they have not always completed each step correctly. In our mark schemes you will find this instruction to markers: When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer. You will often find marks come in pairs, with a method mark followed by an accuracy mark. For example: Marking Instruction AO Marks Differentiates (at least one term correct) AO1.1a M1 Correctly differentiates AO1.1b A1 The first mark is a method mark given for knowing that the correct next step is to differentiate and is given for knowing what to do, even if the maths is not fully correct. The second mark is an accuracy mark and is given for knowing what to do and doing it correctly. The method mark, if given, will follow through on longer questions.

## Here are the important features:

- our marking instructions focus on the mathematical principles being assessed, without being overprescriptive of the techniques or methods to use
- where marks are awarded for accuracy (A), a high proportion allow 'follow through' (ft). This means students can still receive credit after an incorrect result if the next step has been completed successfully. - a high proportion of marks are 'method' marks (M).
- the 'typical solution' on the right-hand side of the mark scheme shows what a very good solution could look like but doesn't describe what students must do. If a student has used another method, sensible application of the mark scheme should be possible to give appropriate credit.
- reasoning (R) marks are awarded for communicating mathematically and applying mathematical principles. They could be given for correct and fully justified work, or for reasoning what the next step needs to be in solving a problem.
- explanation (E) marks are for saying what it is that they are doing or thinking
- you'll see how the assessment objectives are assessed, with each mark allocated an AO


## KS5 Mathematics

- we won't always need a particular method to be used in a student's solution for them to gain marks, however we also respect that some parts of the specification require a certain method to be used and we have to assess this specifically.

Evidence of Work
It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.

Richard P. Feynman

## Class Work and Independent Work

Theory
You will need to organise your notes in a folder that you will keep in either an electronic folder or a paper folder. It's your responsibility to keep the notes and have them organised. Good organisation involves using dates, titles and subtitles and keeping up to date with deadlines and sources of information.

## Problem Solving

This is a crucial part of your work. Solving problems is an application of theory into practice. You are expected to solve problems in class and at home. You are expected to follow certain guidelines and structure and communicate your solutions to reasoning and problem-solving tasks clearly, by showing the working outs to your calculations and answering the questions using good English structure.

## Independent work

The KS5 courses are designed for you to study 10 hours weekly. You will have 5 hours in the classroom with a teacher and 5 hours outside the classroom. It is expected of you to follow your teacher's guidance on how to complete the independent work and to submit the homework on time. It is also expected of you to ask for help when/if you need it and to engage with your learning. In addition, you will be set independent tasks which need to be completed before the deadline from various platforms, such as: Integral Maths and 'My maths'.

## Assessments

There are two possible outcomes: if the result confirms the hypothesis, then you've made a measurement. If the result is contrary to the hypothesis, then you've made a discovery.

Enrico Fermi

Marking of work will take a formative and summative approach with the aim of quality comments on 'closing the gap' on understanding.

## KS5 Mathematics

## Interim:

Formal assessments will take place during different times in the academic year. At the end of each half term there will be mini-mock exams, at the end of each topic (roughly every 2 weeks) there will be end of topic assessments. For AS Mathematics, formal PPE exams will be at the end of your first year; for A-Level Mathematics PPEs will be in November and in the Spring term.

Classwork: Exercise books will be marked once a term with written feedback provided. Also, teachers will be conducting 'live' marking during lessons using red pens and the stamp-written "Verbal feedback".

## Official External exams:

Exams are taken once a year in May/June time. You must ensure that you have sufficient time for revision at home.

## Fun reading list for KS5 mathematics

Even if there is only one possible unified theory, it is just a set of rules and equations. What is it that breathes fire into the equations and makes a universe for them to describe?

Stephen Hawking

These recommendations of reading are for students who enjoy maths and are curious about its applications and history.

Acheson, D.
Clegg, B.
Allenby, RBJT
Courant, Robbins and Stewart
Devlin, K
Maor, E
Stewart, I
Hardy, GH
Hodges, A
Kanigel, R
Eastaway \& Wyndhams

1089 and all that
A brief history of infinity
Numbers and proof
What is Mathematics
Mathematics: The new golden age
' $e$ ' the story of a number
From here to eternity
A mathematician's apology
Alan Turing: The enigma
The man who knew infinity
Why do buses come in threes

## Command and Key words for KS5 mathematics

These action verbs indicate the depth of treatment required for a given assessment statement. These verbs will be used in examination questions and so it is important that students are familiar with the following definitions.

| Define | the precise meaning of a word or phrase as concisely as possible. |
| :---: | :---: |
| Draw | represent by means of pencil lines (add labels unless told not to do so). |
| List | give a sequence of names or other brief answers with no elaboration, each one clearly separated from the others. |
| Measure | find a value for a quantity. |
| State | give a specific name, value or other brief answer (no supporting argument or calculation is necessary). |
| Annotate | add brief notes to a diagram, drawing or graph. |
| Apply | use an idea, equation, principle, theory or law in a new situation. |
| Calculate | find an answer using mathematical methods (show the working unless instructed not to do so). |
| Compare | give an account of similarities and differences between two (or more) items, referring to both (all) of them throughout (comparisons can be given using a table). |
| Describe | give a detailed account, including all the relevant information. |
| Distinguish | give the differences between two or more different items. |
| Estimate | find an approximate value for an unknown quantity, based on the information provided and scientific knowledge. |
| Identify | find an answer from a number of possibilities. |
| Outline | give a brief account or summary (include essential information only). |
| Analyse | interpret data to reach conclusions. |
| Construct | represent or develop in graphical form. |
| Deduce | reach a conclusion from the information given. |
| Derive | manipulate a mathematical equation to give a new equation or result. |
| Design | produce a plan, object, simulation or model. |
| Determine | find the only possible answer. |
| Discuss | give an account including, where possible, a range of arguments, assessments of the relative importance of various factors or comparisons of alternative hypotheses. |
| Evaluate | assess the implications and limitations. |
| Explain | give a clear account including causes, reasons or mechanisms. |
| Predict | give an expected result. |
| Solve | obtain an answer using algebraic and/or numerical methods. |
| Suggest | propose a hypothesis or other possible answer. |
| Hypothesise | write a testable statement. |

## KS5 Mathematics

The tables below set out the notation that must be used by the AS and A-level Mathematics specification. Students will be expected to understand this notation without need for further explanation. AS students will be expected to understand notation that relates to AS content and will not be expected to understand notation that relates only to A-level content.

| Set Notation |  |  |
| :--- | :--- | :--- |
| 1.1 | $\in$ | is an element of |
| 1.2 | $\notin$ | is not an element of |
| 1.3 | $\subseteq$ | is a subset of |
| 1.4 | $\subset$ | is a proper subset of |
| 1.5 | $\left\{x_{1}, x_{2}, \ldots\right\}$ | the set with elements $x_{1}, x_{2}, \ldots$ |
| 1.6 | $\{x: \ldots\}$ | the set of all $x$ such that $\ldots$ |
| 1.7 | $n(A)$ | the number of elements in set $A$ |
| 1.8 | $\varnothing$ | the empty set |
| 1.9 | $\varepsilon$ | the universal set |
| 1.10 | $A^{\prime}$ | the complement of the set $A$ |
| 1.11 | $\mathbb{N}$ | the set of natural numbers, $\{1,2,3, \ldots\}$ |
| 1.12 | $\mathbb{Z}$ | the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$ |
| 1.13 | $\mathbb{Z}^{+}$ | the set of positive integers, $\{1,2,3, \ldots\}$ |
| 1.14 | $\mathbb{Z}_{0}^{+}$ | the set of non-negative integers, $\{0,1,2,3, \ldots\}$ |
| 1.15 | $\mathbb{R}$ | the set of real numbers |

## KS5 Mathematics

| 1.16 | $\mathbb{Q}$ | the set of rational numbers, $\left\{\frac{p}{q}: p \in \mathbb{Z}, q \in \mathbb{Z}^{+}\right\}$ |
| :--- | :--- | :--- |
| 1.17 | $\cup$ | union |
| 1.18 | $\cap$ | intersection |
| 1.19 | $(x, y)$ | the ordered pair $x, y$ |
| 1.20 | $[a, b]$ | the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$ |
| 1.21 | $[a, b)$ | the interval $\{x \in \mathbb{R}: a \leq x<b\}$ |
| 1.22 | $(a, b]$ | the interval $\{x \in \mathbb{R}: a<x \leq b\}$ |
| 1.23 | $(a, b)$ | the open interval $\{x \in \mathbb{R}: a<x<b\}$ |


| Miscellaneous symbols |  |  |
| :--- | :--- | :--- |
| 2.1 | $=$ | is equal to |
| 2.2 | $\neq$ | is not equal to |
| 2.3 | $\equiv$ | is identical to or is congruent to |
| 2.4 | $\approx$ | is approximately equal to |
| 2.5 | $\infty$ | infinity |
| 2.6 | $\propto$ | is proportional to |
| 2.7 | $\therefore$ | therefore |
| 2.8 | $\because$ | because |
| 2.9 | $<$ | is less than |
| 2.10 | $\leq$ | is less than or equal to |
| 2.11 | $>$ | is greater than |
| 2.12 | $\geq$ | is greater than or equal to |
| 2.13 | $p \Rightarrow q$ | $p$ implies $q$ (if $p$ then $q$ ) |
| 2.14 | $p q$ | $p$ is implied by $q$ (if $q$ then $p$ ) |

## KS5 Mathematics

UTC
SWINDON

| 2.15 | $p \Leftrightarrow q$ | $p$ implies and is implied by $q(p$ is equivalent to $q)$ |
| :--- | :--- | :--- |
| 2.16 | $a$ | first term of an arithmetic or geometric sequence |
| 2.17 | $l$ | last term of an arithmetic sequence |
| 2.18 | $d$ | common difference of an arithmetic sequence |
| 2.19 | $r$ | common ratio of a geometric sequence |
| 2.20 | $S_{n}$ | sum to $n$ terms of a series |
| 2.21 | $S_{\infty}$ | sum to infinity of a geometric series |


| Operations |  | $a$ plus $b$ |
| :--- | :--- | :--- |
| 3.1 | $a+b$ | $a$ minus $b$ |
| 3.2 | $a-b$ | $a$ multiplied by $b$ |
| 3.3 | $a \times b, a b, a . b$ | $a$ divided by $b$ |
| 3.4 | $a \div b, \frac{a}{b}$ | $a_{1}+a_{2}+\ldots a_{n}$ |
| 3.5 | $\sum_{i=1}^{n} a_{i}$ | $a_{1} \times a_{2} \times \ldots a_{n}$ |
| 3.6 | $\prod_{i=1}^{n} a_{i}$ | This notation is not required in AS or A-level Mathematics |
| 3.7 | $\sqrt{a}$ | the non-negative square root of $a$ |
| 3.8 | $\|a\|$ | the modulus of $a$ |
| 3.9 | $n!$ | $n$ factorial: $n!=n \times(n-1) \times \ldots \times 2 \times 1, n \in \mathbb{N} ; 0!=1$ |
| 3.10 | $\binom{n}{r},{ }^{n} \mathrm{C}_{r},{ }_{n} \mathrm{C}_{r}$ |  |


| Functions |  | the value of the function f at $x$ |
| :--- | :--- | :--- |
| 4.1 | $\mathrm{f}(x)$ | the function f maps the element $x$ to the element $y$ |
| 4.2 | $\mathrm{f}: x \mapsto y$ | the inverse function of the function f |
| 4.3 | $\mathrm{f}^{-1}$ | the composite function of f and g which is defined by <br> $\mathrm{gf}(x)=\mathrm{g}(\mathrm{f}(x))$ |
| 4.4 | gf | the limit of $\mathrm{f}(x)$ as $x$ tends to $a$ |
| 4.5 | $\lim _{x \rightarrow a} \mathrm{f}(x)$ | an increment of $x$ |
| 4.6 | $\Delta x, \delta x$ | the derivative of $y$ with respect to $x$ |
| 4.7 | $\frac{\mathrm{~d} y}{\mathrm{~d} x}$ | $\frac{\mathrm{~d}^{\mathrm{n}} y}{\mathrm{~d} x^{n}}$ |
| 4.8 | $\mathrm{f}^{\prime}(x), \mathrm{f}^{\prime \prime}(x), \ldots, \mathrm{f}^{(n)}(x)$ | the $n$th derivative of $y$ with respect to $x$ |
| 4.10 | $\dot{x}, \ddot{x}, \ldots$ | the first, second, $\ldots, n$th derivatives of $\mathrm{f}(x)$ with respect to $x$ |
| 4.11 | $\int y \mathrm{~d} x$ | the first, second, ... derivatives of $x$ with respect to $t$ |
| 4.12 | $\int_{a}^{b} y \mathrm{~d} x$ | This notation will not be used in AS and A-level Mathematics |


| Exponentials and logarithmic functions |  |  |
| :--- | :--- | :--- |
| 5.1 | e | base of natural logarithms |
| 5.2 | $\mathrm{e}^{x}, \exp x$ | exponential function of $x$ <br> The notation exp $x$ will not be used in this specification. |
| 5.3 | $\log _{a} x$ | logarithm to the base $a$ of $x$ |
| 5.4 | $\ln x, \log _{\mathrm{e}} x$ | natural logarithm of $x$ |

## KS5 Mathematics

| Trigonometric functions |  |  |
| :---: | :---: | :---: |
| 6.1 | $\left.\begin{array}{l} \sin , \cos , \tan \\ \operatorname{cosec}, \text { sec, cot } \end{array}\right\}$ | the trigonometric functions |
| 6.2 | $\left.\begin{array}{l} \sin ^{-1}, \cos ^{-1}, \tan ^{-1} \\ \arcsin , \arccos , \arctan \end{array}\right\}$ | the inverse trigonometric functions |
| 6.3 | 。 | degrees |
| 6.4 | rad | radians <br> Typically no symbol will be used to denote the use of radian measure. |


| Vectors |  |  |
| :--- | :--- | :--- |
| 9.1 | $\mathbf{a}, \underline{\mathbf{a}}, \mathbf{a}$ | the vector $\mathbf{a}, \mathbf{a}, \mathbf{a}$; these alternatives apply throughout section <br> 9 |
| 9.2 | $\overrightarrow{A B}$ | the vector represented in magnitude and direction by the <br> directed line segment $A B$ |
| 9.3 | $\hat{\mathbf{a}}$ | a unit vector in the direction of a |
| 9.4 | $\mathbf{i}, \mathbf{j}, \mathbf{k}$ | unit vectors in the directions of the Cartesian coordinate axes |
| 9.5 | $\|\mathbf{a}\|, a$ | the magnitude of $\mathbf{a}$ |
| 9.6 | $\|\overrightarrow{A B}\|, A B$ | the magnitude of $\overrightarrow{A B}$ |
| 9.7 | $\left[\begin{array}{l}a \\ b\end{array}\right], a \mathbf{i}+b \mathbf{j}$ | column vector and corresponding unit vector notation <br> We will use square brackets, but round brackets are perfectly <br> acceptable. |
| 9.8 | $\mathbf{r}$ | position vector |
| 9.9 | $\mathbf{s}$ | displacement vector |
| 9.10 | $\mathbf{v}$ | velocity vector |
| 9.11 | $\mathbf{a}$ | acceleration vector |


| Probability and statistics |  |  |
| :---: | :---: | :---: |
| 11.1 | $A, B, C$, etc | events |
| 11.2 | $A \cup B$ | union of the events $A$ and $B$ |
| 11.3 | $A \cap B$ | intersection of the events $A$ and $B$ |
| 11.4 | $\mathrm{P}(A)$ | probability of the event $A$ |
| 11.5 | $A^{\prime}$ | complement of the event $A$ |
| 11.6 | $\mathrm{P}(A \mid B)$ | probability of the event $A$ conditional on the event $B$ |
| 11.7 | $X, Y, R$, etc | random variables |
| 11.8 | $x, y, r$, etc | values of the random variables $X, Y, R$, etc |
| 11.9 | $x_{1}, x_{2}, \ldots$ | values of observations |
| 11.10 | $f_{1}, f_{2}, \ldots$ | frequencies with which the observations $x_{1}, x_{2}, \ldots$ occur |
| 11.11 | $\mathrm{p}(x), \mathrm{P}(X=x)$ | probability function of the discrete random variable $X$ |
| 11.12 | $\mathrm{p}_{1}, \mathrm{p}_{2}, \ldots$ | probabilities of the values $x_{1}, x_{2}, \ldots$ of the discrete random variable $X$ |
| 11.15 | $\sim$ | has the distribution |
| 11.16 | $\mathrm{B}(n, p)$ | binomial distribution with parameters $n$ and $p$, where $n$ is the number of trials and $p$ is the probability of success in a trial |
| 11.17 | $q$ | $q=1-p$ for binomial distribution |
| 11.18 | $\mathrm{N}\left(\mu, \sigma^{2}\right)$ | Normal distribution with mean $\mu$ and variance $\sigma^{2}$ |
| 11.19 | $Z \sim \mathrm{~N}(0,1)$ | standard Normal distribution |
| 11.20 | $\phi$ | probability density function of the standardised Normal variable with distribution $N(0,1)$ <br> Not required in AS and A-level Mathematics. |

## KS5 Mathematics

UTC
SWINDON

| 11.22 | $\mu$ | population mean |
| :--- | :--- | :--- |
| 11.23 | $\sigma^{2}$ | population variance |
| 11.24 | $\sigma$ | population standard deviation |
| 11.25 | $\bar{x}$ | sample mean |
| 11.28 | $\mathrm{H}_{0}$ | null hypothesis |
| 11.29 | $\mathrm{H}_{1}$ | alternative hypothesis |
| 11.30 | $r$ | product moment correlation coefficient for a sample |
| 11.31 | $\rho$ | product moment correlation coefficient for a population |


| Mechanics |  |  |
| :--- | :--- | :--- |
| 12.1 | kg | kilogram(s) |
| 12.2 | m | metre(s) |
| 12.3 | km | kilometre(s) |
| 12.4 | $\mathrm{~m} / \mathrm{s}, \mathrm{m} \mathrm{s}^{-1}$ | metre(s) per second (velocity) |
| 12.5 | $\mathrm{~m} / \mathrm{s}^{2}, \mathrm{~m} \mathrm{~s}^{-2}$ | metre(s) per second per second (acceleration) |
| 12.6 | $F$ | force or resultant force |
| 12.7 | N | newton |
| 12.8 | Nm | newton metre (moment of force) |
| 12.9 | $t$ | time |
| 12.10 | $s$ | displacement |
| 12.11 | $u$ | nitial velocity |
| 12.12 | $v$ | velocity or final velocity |
| 12.13 | $a$ | acceleration |
| 12.14 | $g$ | acceleration due to gravity |
| 12.15 | $\mu$ | coefficient of friction |

## KS5 Mathematics

## The Bridge to KS5 Maths

## Problem Solving



Please attempt the following tasks before starting your KS5 maths course.

## 1 <br> Solving quadratic equations

## Question 1

A number and its reciprocal add up to $\frac{26}{5}$.
Form and solve an equation to calculate the number.

## Question 2

The diagram shows a trapezium.

Diagram NOT accurately drawn

All the measurements are in centimetres.
The area of the trapezium is $16 \mathrm{~cm}^{2}$.

a) Show that $2 x^{2}+5 x-16=0$
b) Work out the value of $x$ to 1 decimal place.

$$
\begin{equation*}
x=. \tag{2}
\end{equation*}
$$

## Question 3

Two numbers have a product of 44 and a mean of 7.5.
Use an algebraic method to find the numbers.
You must show all of your working.

## KS5 Mathematics

## 2 Changing the subject

## Question 1

The surface gravity of a planet is given by $g=\frac{G M}{r^{2}}$ where
$M=$ Mass of the planet
$r=$ radius of the planet
$\mathrm{G}=$ gravitational constant $=6.67 \times 10^{-11}$
The surface gravity of Earth is $9.807 \mathrm{~m} / \mathrm{s}^{2}$ and the mass of Earth is $5.98 \times 10^{24} \mathrm{~kg}$.
Find the radius of Earth in kilometres correct to 3 significant figures.

## Question 2

In a parallel circuit, the total resistance is given by the formula $\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$
Make $R_{1}$ the subject of the formula

## Question 3

Show that $\frac{1}{\frac{1}{x}+1}=\frac{x}{x+1}$

## 3 Simultaneous equations

## Question 1

Sarah intended to spend $£ 6.00$ on prizes for her class but each prize cost her 10 p more than expected, so she had to buy 5 fewer prizes.
Calculate the cost of each prize.

## Question 2

Arthur and Florence are going to the theatre.
Arthur buys 6 adult tickets and 2 child tickets and pays $£ 39$.
Florence buys 5 adult tickets and 3 child tickets and pays $£ 36.50$.
Work out the costs of both adult and child tickets.

## KS5 Mathematics

## Question 1

Calculate the area of each shape giving your answers in the form $a+b \sqrt{2}$
a)
$11-\sqrt{ } 2$

b)


## Question 2

Colin has made several mistakes in his 'simplifying surds' homework. Explain his error and give the correct answer.
i) $\quad 4 \sqrt{3} \times 5 \sqrt{12}=20 \sqrt{36}$

## KS5 Mathematics

## Question 3

The area of a triangle is $20 \mathrm{~cm}^{3}$. The length of the base is $\sqrt{8} \mathrm{~cm}$. Work out the perpendicular height giving your answer as a surd in its simplest form.

## Question 1

Lowenna says that $27^{-1 / 3} \times 64^{2 / 3}=48$
Is Lowenna correct? You must show all of your working.

## Question 2

Which one of these indices is the odd one out? Circle your answer and give reasons for your choice.
$16^{-\frac{1}{4}}$
$64^{-\frac{1}{2}}$
$8^{-\frac{1}{3}}$

## Question 3

Find values for $a$ and $b$ that make this equation work
$a^{\frac{1}{2}}=b^{\frac{1}{3}}$

## Question 4

i) Write 25 as a power of 125
ii) Write 4 as a power of 32
iii) Write 81 as a power of 27

## 6 <br> Properties of Lines

## Question 1

(a) Write down the gradient of the line $2 y-4 x=5$.
(b) Write down the equation of a line parallel to $3 y=7-4 x$.
(c) Write down the equation of a line with gradient $1 / 2$ and $y$-intercept of 6 .

## Question 2

Here is the profile of the first half of a fell running race.

(a) Work out the approximate gradient of the race from the start to Mad Major's Grave
(b) The most dangerous part of the race is from Mad Major's Grave to the Footbridge. Why do you think this might be?
(c) Work out an estimate for the average ascent for the first four uphill sections of the race.

## KS5 Mathematics

## Question 3

Here is a graph used to convert degrees Celsius (C) and degrees Fahrenheit (F).


The equation of the straight line is given by $F=m C+a$ Calculate the values of $m$ and $a$
$\square$

## KS5 Mathematics

## 7 <br> Sketching curves

## Question 1

Sketch the graph of $f(x)=x^{2}+5 x-6$, showing the co-ordinates of the turning point and the coordinates of any intercepts with the coordinate axes.


## Question 2

a) On the axes sketch the graph of $y=\frac{3}{x}$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.


## KS5 Mathematics

b) On the axes sketch the graph of $y=x^{3}-5$ showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.


## KS5 Mathematics

## 8 Transformation of functions

## Question 1

Here is a sketch of $f(x)$.
The coordinates of P are ( $0,-2$ )
Sketch the graphs after the following translations and reflections, and state the coordinates of $P^{\prime}$ :
a) $g(x)=f(x)+1$
b) $h(x)=f(x-2)$
c) $j(x)=-f(x)$
d) $k(x)=f(-x)$

## Question 2



The graph of $y=f(x)$ is shown below.


Below each sketch, write down the equation of the transformed graph



## KS5 Mathematics

$$
y=. . . . . . . . . . . . . . . . . . . . ~ \quad y=. .
$$

## Question 3

The equation of a curve is $y=f(x)$ where $f(x)=x^{2}-4 x+5$
C is the minimum point of the curve.
(a) Find the coordinates of C after the transformation $f(x+1)+2$.
(b) Hence, or otherwise, determine if $f(x-3)-1=0$ has any real roots. Give reasons for your answer.

## $9 \quad$ Pythagoras' theorem and Trigonometric ratios

## Question 1

ABCDEFGH is a cuboid
$\mathrm{AE}=5 \mathrm{~cm}$
$\mathrm{AB}=6 \mathrm{~cm}$
$B C=9 \mathrm{~cm}$


Diagram NOT drawn accurately
(a) Calculate the length of AG. Give your answer correct to 3 significant figures.
(b) Calculate the size of the angle between AG and the face ABCD.

Give your answer correct to 1 decimal place.

## Question 2

A piece of land is the shape of an isosceles triangle with sides $7.5 \mathrm{~m}, 7.5 \mathrm{~m}$ and 11 m .
Turf can be bought for $£ 11.99$ per $5 \mathrm{~m}^{2}$ roll.
How much will it cost to turf the piece of land?

## Question 3

Ben is 1.62 m tall.
The tent he is considering buying is a square based pyramid.
The length of the base is 3.2 m .
The poles $\mathrm{AE}, \mathrm{CE}, \mathrm{AE}$ and BE are 2 m long.


Ben wants to know if he will be able to stand up in the middle of the tent. Explain your answer clearly.
$\qquad$

## 10 Sine / Cosine Rule

## Question 1

Plane A is flying directly toward the airport which is 20 miles away. The pilot notice a second plane, $B, 45^{\circ}$ to her right. Plane $B$ is also flying directly towards the airport. The pilot of plane $B$ calculates that plane $A$ is $50^{\circ}$ to his left. Based on that information how far is plane $B$ from the airport? Give your answer to 3 significant figures.

## Question 2

Two ships, A and B, leave the same port at the same time.
Ship A travels at $35 \mathrm{~km} / \mathrm{h}$ on a bearing of $130^{\circ}$.
Ship B travels at $25 \mathrm{~km} / \mathrm{h}$ on a bearing of $120^{\circ}$.
Calculate how far apart the ships are after 1 hour.
Give your answer correct to two decimal places.

## Question 3

A farmer has a triangular field. He knows one side measures 450 m and another 320 m . The angle between these two sides measures $80^{\circ}$. The farmer wishes to use a fertiliser that costs $£ 3.95$ per container which covers $1500 \mathrm{~m}^{2}$. How much will it cost to use the fertiliser on this field?

## 11 Inequalities

## Question 1

A new cylindrical tube of snacks is being designed so that its height is 3 times its radius and its volume must be less than 20 times its radius.
Create an inequality and find possible values for the radius.

## Question 2

A base jumper is going to jump off a cliff that is 50 m tall, the distance she travels downwards is given by the equation

$$
\begin{array}{lll}
\mathrm{d}=4.9 \mathrm{t}^{2} & \text { where } & \mathrm{t}=\text { time of flight } \\
\text { and } & \mathrm{d}=\text { distance travelled }
\end{array}
$$

A video camera is set-up to film her between 20 m and 10 m above the ground. Calculate the time period after the jumper jumps that filming taking place.

## Question 3

The total volume of the box is less than 1 litre. Given that all lengths are in cm and that x is an integer, $x$ Show that the longest side is less than 18 cm .


## 12 <br> Algebraic proof

## Question 1

Katie chooses a two-digit number, reverses the digits, and subtracts the smaller number from the larger.

For example
$42-24=18$
She tries several different numbers and finds the answer is never a prime number.
Prove that Katie can never get an answer that is a prime number.

## Question 2

Here are the first 5 terms of an arithmetic sequence

$$
\begin{array}{lllll}
1 & 6 & 11 & 16 & 21
\end{array}
$$

Prove that the difference between the squares of any 2 terms is always a multiple of 5.

## KS5 Mathematics

## 13 Vectors

## Question 1

OAB is a triangle
$\overrightarrow{\mathrm{OA}}=\mathbf{a}$ and $\overrightarrow{\mathrm{OB}}=\mathbf{b}$
(a) Find the vector $\overrightarrow{A B}$ in terms of $\mathbf{a}$ and $\mathbf{b}$


Diagram NOT drawn accurately
$P$ is the point on $\overrightarrow{\mathrm{AB}}$ such that $\mathrm{AP}: \mathrm{PB}=3: 2$
(b) Show that $\overrightarrow{O P}=\frac{1}{5}(2 \mathbf{a}+3 \mathbf{b})$

## Question 2

OABC is a parallelogram.
$X$ is the midpoint of $O B$
$\overrightarrow{O A}=\mathbf{a}$ and $\overrightarrow{O C}=\mathbf{c}$


Diagram NOT
drawn
accurately
(a) Find the vector $\overrightarrow{O X}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

## KS5 Mathematics

(b) Find the vector $\vec{X} C$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

## Question 3

PQRS is a parallelogram. $M$ is the midpoint of $R S$ N is the midpoint of QR
$\overrightarrow{P Q}=2 \mathbf{a}$
$\overrightarrow{\mathrm{PS}}=2 \mathrm{~b}$


Diagram NOT drawn accurately

Use vectors to proof that the line segments SQ and MN are parallel.
$\square$

## 14

Probability

## Question 1

Max has an empty box.
He puts some red counters and some blue counters into the box.
The ratio of the number of red counters to the number of blue counters is $1: 3$.
Julie takes at random 2 counters from the box.
The probability that she takes 2 red counters is $\frac{19}{316}$.
How many red counters did Max put in the box?

## KS5 Mathematics

## Question 2

The Venn diagram shows the ice-cream flavours chosen by a group of 44 children at a party.
The choices are strawberry (S), choc-chip (C) and toffee (T).
A child is picked at random.


Work out:
(a) P(S)
(b) $\mathrm{P}(\mathrm{T} \cup \mathrm{C} \mid \mathrm{C})$
(c) $\mathrm{P}(\mathrm{C} \mid \mathrm{S}$ U T)

## KS5 Mathematics

## 15 Statistics

## Question 1

The table and histogram show the weights of some snakes.

| Weight, grams |  | Frequency |
| :---: | :---: | :---: |
| $250<x \leq 300$ | 60 |  |
| $300<x \leq 325$ | 25 |  |
| $325<x \leq 350$ | 40 |  |
| $350<x \leq 450$ | 35 |  |
| $450<x \leq 600$ | 40 |  |
|  | Total | 200 |
|  |  |  |


(a) Use the information to complete the histogram
(3)
(b) Calculate an estimate for the median

## Question 2

Sarah played 15 games of netball. Here are the number of goals she scored in each game.

| 17 | 17 | 17 | 18 | 19 | 20 | 21 | 22 | 24 | 25 | 25 | 26 | 28 | 28 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | 28

a) Draw a boxplot to show this information

a) Lucy plays in the same 15 games of netball. The median number of points Lucy scores is 24. The interquartile range of these points is 10 and the range of these points is 17 .

Who is the better player, Sarah or Lucy?
You must give a reason for your answer.


